Circuit
Quantum Electrodynamics

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Atom in a Cavity

Consider only two levels of atom, with energy separation $\hbar \Omega$

Atom drifts through electromagnetic resonant cavity with very high $Q$

Jaynes-Cummings Hamiltonian:

$$H = \hbar \omega_r \left( a^\dagger a + \frac{1}{2} \right) + \frac{\hbar \Omega}{2} \sigma^z + \hbar g (a^\dagger \sigma^- + \sigma^+ a) + H_\kappa + H_\gamma.$$ 

coupling strength $g = \varepsilon_{\text{rms}} d / \hbar$

cavity decay rate $\kappa = \omega_r / Q$

decay rate to non-cavity modes - $\gamma$

Strong Coupling limit

$$g \gg \kappa, \gamma$$

Needed:

- High $Q$
- Large $d$
Electropolished superconducting Nb cavity

50 mm diameter and a 40 mm radius of curvature

Nice presentation at: http://www.lkb.ens.fr/recherche/qedcav/english/englishframes.html
Energy Eigenstates

\[ H = \hbar \omega_r \left( a^\dagger a + \frac{1}{2} \right) + \frac{\hbar \Omega}{2} \sigma^z + \hbar g (a^\dagger \sigma^- + \sigma^+ a) + H_A + H_B \]

Neglect damping for the moment, exact diagonalization gives energy eigenstates:

\[ \Delta \equiv \Omega - \omega_r \text{ the atom-cavity detuning.} \]

\[ E_{\uparrow,0} = -\frac{\hbar \Delta}{2}, \quad \theta_n = \frac{1}{2} \tan^{-1} \left( \frac{2g\sqrt{n+1}}{\Delta} \right), \]

\[ |\pm, n\rangle = \cos \theta_n |\downarrow, n\rangle + \sin \theta_n |\uparrow, n+1\rangle, \]

\[ |\mp, n\rangle = -\sin \theta_n |\downarrow, n\rangle + \cos \theta_n |\uparrow, n+1\rangle, \]

\[ E_{\pm, n} = (n + 1)\hbar \omega_r \pm \frac{\hbar}{2} \sqrt{4g^2(n + 1) + \Delta^2}, \]

For zero detuning the degeneracy of the photon states with the atom state is lifted by the coupling. These “dressed states” are “maximally entangled” atom – field states

\[ |\pm, 0\rangle = (|\uparrow, 1\rangle \pm |\downarrow, 0\rangle) / \sqrt{2} \]

Blais et. al, Phys. Rev. A, 2004
Spectrum

Vacuum Rabi flopping:

\[ |\downarrow,0\rangle \rightarrow |\uparrow,1\rangle \]

0 photons in cavity
atom in GS
1 photon in cavity

Level splitting depends on number of photons in cavity
\[ 2g\sqrt{n+1} \]

Blais et. al, Phys. Rev. A, 2004
Now consider damping: excitation is $\frac{1}{2}$ photon, $\frac{1}{2}$ atom $\Rightarrow$ decay rate: $\frac{\kappa + \gamma}{2}$

In *strong coupling limit* there is a splitting of cavity resonance which can be resolved because:

$$g = \frac{\mathcal{E}_{\text{rms}} d}{\hbar} \gg \kappa, \gamma$$

Blais et. al, Phys. Rev. A, 2004
Coplanar Waveguide (CPW) really a transmission line

Top View

Superconducting film
Gap in SC film

cut

Transverse view on cut

Electrostatic Potential
CP box in Microstrip line cavity

At a resonant frequency of 10 GHz ($h\nu/k_B \sim 0.5$ K)

$$V_{\text{rms}}^0 \sim \sqrt{\hbar \omega_r / cL} \sim 2 \mu V \quad \varepsilon_{\text{rms}} \sim 0.2 \text{ V/m}$$

Very small effective volume, $\sim 10^{-5}$ cubic wavelengths

Blais et. al, Phys. Rev. A, 2004
Cooper Pair Box as TLS

\[ H_Q = 4E_c \sum_{N} (N - N_g)^2 \langle N \rangle \langle N \rangle - \frac{E_J}{2} \sum_{N} (\langle N + 1 \rangle \langle N \rangle + \text{H.c.}), \]

\[ E_c = \frac{e^2}{2C_\Sigma} \]

\[ E_J = \frac{\hbar}{2e} I_c = \frac{\hbar \pi \Delta(T)}{4e^2 R_N} \]

\[ H_Q = -\frac{E_{el}}{2} \sigma^z - \frac{E_J}{2} \sigma^x, \]

\[ E_{el} = 4E_C(1 - 2N_g) \]

In the charge regime \( 4E_c \gg E_J \)

\[ \text{SQUID junction} \rightarrow \text{tune } E_J \]

\[ E_J \cos(\pi \Phi_{ext}/\Phi_0) / 2 \]
Map to James-Cummings Hamiltonian

\[ H = \hbar \omega_x \left( a^\dagger a + \frac{1}{2} \right) + \frac{\hbar \Omega}{2} \sigma^z + \hbar g (a^\dagger \sigma^- + \sigma^+ a) + H_\kappa + H_\gamma \]

\[ \Omega = \frac{E_J}{\hbar} \quad g = \frac{\beta e}{\hbar} \sqrt{\frac{\hbar \omega_r}{c L}} \quad \beta \equiv \frac{C_g}{C_\Sigma} \]

Very large effective dipole moment:

\[ d \equiv \frac{\hbar g}{\mathcal{E}_{\text{rms}}} \sim 2 \times 10^4 \text{ atomic units (ea}_0) \]

Blais et. al, Phys. Rev. A, 2004
Atom vs. Circuit implementation

Atomic Physics:
Measure shift of atom level which drifts through cavity and infer the state of photons in the cavity.

Circuit QED:
Directly measure transmission of cavity and observe splitting of cavity resonance.

“Atom” replaced by a superconducting circuit with quantized energy.

Circuit does not drift, no transit time.

Circuit two-level-system can be tuned with external voltage and current.

Blais et. al, Phys. Rev. A, 2004
Comparison

TABLE I. Key rates and CQED parameters for optical [2] and microwave [3] atomic systems using 3D cavities, compared against the proposed approach using superconducting circuits, showing the possibility for attaining the strong cavity QED limit ($n_{\text{Rabi}} \gg 1$). For the 1D superconducting system, a full-wave ($L = \lambda$) resonator, $\omega_0/2\pi = 10$ GHz, a relatively low $Q$ of $10^4$, and coupling $\beta = C_g/C_\Sigma = 0.1$ are assumed. For the 3D microwave case, the number of Rabi flops is limited by the transit time. For the 1D circuit case, the intrinsic Cooper-pair box decay rate is unknown; a conservative value equal to the current experimental upper bound $\gamma \approx 1/(2\mu s)$ is assumed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>3D optical</th>
<th>3D microwave</th>
<th>1D circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonance or transition frequency</td>
<td>$\omega_0/2\pi, \Omega/2\pi$</td>
<td>350 THz</td>
<td>51 GHz</td>
<td>10 GHz</td>
</tr>
<tr>
<td>Vacuum Rabi frequency</td>
<td>$g/\pi, g/\omega_0$</td>
<td>220 MHz, $3 \times 10^{-7}$</td>
<td>47 kHz, $1 \times 10^{-7}$</td>
<td>100 MHz, $5 \times 10^{-3}$</td>
</tr>
<tr>
<td>Transition dipole</td>
<td>$d/ea_0$</td>
<td>$\sim 1$</td>
<td>$1 \times 10^3$</td>
<td>$2 \times 10^4$</td>
</tr>
<tr>
<td>Cavity lifetime</td>
<td>$1/\kappa, Q$</td>
<td>10 ns, $3 \times 10^7$</td>
<td>1 ms, $3 \times 10^8$</td>
<td>160 ns, $10^4$</td>
</tr>
<tr>
<td>Atom lifetime</td>
<td>$1/\gamma$</td>
<td>61 ns</td>
<td>30 ms</td>
<td>2 $\mu$s</td>
</tr>
<tr>
<td>Atom transit time</td>
<td>$t_{\text{transit}}$</td>
<td>$\geq 50 \mu s$</td>
<td>100 $\mu s$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Critical atom number</td>
<td>$N_0 = 2\gamma\kappa/g^2$</td>
<td>$6 \times 10^{-3}$</td>
<td>$3 \times 10^{-6}$</td>
<td>$\leq 6 \times 10^{-5}$</td>
</tr>
<tr>
<td>Critical photon number</td>
<td>$m_0 = \gamma^2/2g^2$</td>
<td>$3 \times 10^{-4}$</td>
<td>$3 \times 10^{-8}$</td>
<td>$\leq 1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Number of vacuum Rabi flops</td>
<td>$n_{\text{Rabi}} = 2g/(\kappa + \gamma)$</td>
<td>$\sim 10$</td>
<td>$\sim 5$</td>
<td>$\sim 10^2$</td>
</tr>
</tbody>
</table>

- Cooper Pair Box does not drift, stays in place $t_{\text{transit}} = \infty$
- Examine one (and the same) quantum system
- Tune parameters of CPB Hamiltonian with external gate voltage and magnetic flux

Blais et. al, Phys. Rev. A, 2004
Experiment 1 by Yale Group

Nb cavity on Si/SiO2 substrate, length 24 cm,
Schematic of measurement

\[ Q = \frac{\nu_r}{\delta \nu_r} \approx 10^4. \]

Wallraff et. al, NATURE, 2004
Mapping tuning conditions

Phase shift of transmitted microwaves at frequency $\omega_r$

Density plot of measured phase shift

Wallraff et. al, NATURE, 2004
Vacuum Rabi splitting

By fitting the split cavity resonance, they can determine the mean number of thermal photons in the cavity

Wallraff et. al, NATURE, 2004
Atom – Cavity detuning

\[ \omega_r = \Omega - \Delta \]

Large detuning: \( \frac{g}{\Delta} \ll 1 \)

Atom transition is ac Stark/Lamb shifted by \( (g/\Delta)^2 (n + 1/2) \)

Atom “pulls” cavity resonant frequency: \( \omega_r \Rightarrow \omega_r \pm \frac{g^2}{\kappa \gamma} \)

Blais et. al, Phys. Rev. A, 2004
Measuring qubit with cavity

Blais et. al, Phys. Rev. A, 2004
Experiment 2 by Yale Group

• Work in non-resonant regime \(|\Delta| = |\omega_a - \omega_r| > g\)

• Use pulling of cavity resonance to read out state of TLS (qubit)

• Apply microwave pulse at \(\omega_a\) to prepare qubit state (Rabi oscillations)

Experimental schematic

“meter” response after Rabi pulse

Rabi oscillations with unit visibility in readout

Quantum dot in Photonic Crystal Cavity

Yoshie et. al, NATURE 2004
Vacuum Rabi splitting of cavity resonance

Yoshie et. al, NATURE 2004
Bibliography


• Web site of [LABORATOIRE KASTLER BROSSEL](http://www.lkb.ens.fr/recherche/qedcav/english/englishframes.html) with info on Atom-Cavity QED, link to publication list
